

if, $x + \frac{1}{x} = 1$ तो $x^3 = -1$



CONCEPT



तों

If, $x + \frac{1}{x} = -1$

OR

$$x^2 + x + 1 = 0$$

$$\rightarrow x^3 = 1$$

$$x + \frac{1}{x} = -1$$

$$(x-1) \times (x^2 + x + 1) = 0 \times (x-1)$$

$$x^3 - 1 = 0$$

$$\Rightarrow x^3 = 1$$

41. If $\frac{x}{5\sqrt{6}} + 1 = \frac{-5\sqrt{6}}{x}$, then find the value of $x^5 - 750\sqrt{6}x^2 + 1 = ?$

यदि $\frac{x}{5\sqrt{6}} + 1 = \frac{-5\sqrt{6}}{x}$, तो $x^5 - 750\sqrt{6}x^2 + 1$ का मान ज्ञात कीजिए?

[A] $25\sqrt{6}$

✓ [B] 1

[C] 0

[D] $1 + 5\sqrt{6}$

~~$x^5 - x^5 + 1$~~

$$\left(\frac{x}{5\sqrt{6}}\right) + \left(\frac{5\sqrt{6}}{x}\right) = -1$$

$$\Rightarrow \left(\frac{x}{5\sqrt{6}}\right)^3 = 1$$

$$x^3 = 750\sqrt{6}$$

42. If $\frac{a}{b} + \frac{b}{a} = -1$ and $a - b = 2$, then the value of $a^3 - b^3$ is:
यदि $\left(\frac{a}{b}\right) + \left(\frac{b}{a}\right) = -1$ और $a - b = 2$ है, तो $a^3 - b^3$ का मान ज्ञात करें।
 $= 0$

(SSC CGL 2022)

[A] 0 ✓

[B] $\frac{1}{2}$

[C] 1

[D] -1

$$\left(\frac{a}{b}\right)^3 = 1$$

$$a^3 = b^3$$



43. If $5x + \frac{1}{2x} = \sqrt{5}$, then find $50x^4 + \frac{1}{100x^4}$?

यदि $5x + \frac{1}{2x} = \sqrt{5}$ है, तो $50x^4 + \frac{1}{100x^4}$ ज्ञात कीजिये?

[A] -1

[B] -1/2

[C] -4/3

[D] -3/2

$$-\frac{1}{2} - 1 = -\frac{3}{2}$$

$$25x^2 + \frac{1}{4x^2} = \sqrt{5} \quad \text{---} \times 5 \times \frac{1}{2}$$

$$\frac{100x^4 + 1}{4x^2} = 0$$

$$\Rightarrow 100x^4 + 1 = 0 \Rightarrow 100x^4 = -1$$

44. If $x^4 + y^4 = x^2y^2$, then find $x^6 + y^6$?

यदि $x^4 + y^4 = x^2y^2$, तो $x^6 + y^6$ ज्ञात करें?

(CPO 2023)

[A] 1

[B] 3

[C] 2

[D] 0



$$(x^2 + y^2)^2 = x^4 + y^4 + 2x^2y^2$$

$$(x^2 + y^2)^2 - (xy)^2 = x^4 + y^4 + 2x^2y^2$$

$$(x^2 + y^2 + xy) \cdot (x^2 + y^2 - xy) = x^4 + y^4 + 2x^2y^2$$

$\downarrow$ A माना $\downarrow$ B माना

$$x^2 + y^2 + xy = A$$

$$x^2 + y^2 - xy = B$$

$$(x^2 + y^2) = \frac{A+B}{2}$$

$$xy = \frac{A-B}{2}$$





CONCEPT


$$(x^2 + y^2)^2 = x^4 + y^4 + 2x^2y^2$$

$$x^4 + y^4 + x^2y^2 = (x^2 + y^2)^2 - (xy)^2$$

$$x^4 + y^4 + x^2y^2 = (x^2 + y^2 \textcolor{green}{+} xy)(x^2 + y^2 \textcolor{green}{-} xy)$$

$$(x^2 + y^2) = \frac{A + B}{2} \quad \& \quad xy = \frac{A - B}{2}$$

QUESTION

$$x^4 + x^2y^2 + y^4 = (x^2 + xy + y^2)(x^2 - xy + y^2)$$

If, $x^4 + x^2y^2 + y^4 = 189$ & $x^2 + xy + y^2 = 21$

(a) $x^2 + xy + y^2 = 21$ (A)
 $x^2 - xy + y^2 = \frac{189}{21} = 9$ (B)

(b) $x^2 + y^2 = \frac{21+9}{2} = 15$

(c) $xy = \frac{21-9}{2} = 6$

(d) $\frac{x}{y} + \frac{y}{x} = \frac{x^2+y^2}{xy} = \frac{15}{6} = \frac{5}{2}$

(e) $\frac{x^2}{y^2} + \frac{y^2}{x^2} = \left(\frac{5}{2}\right)^2 - 2 = \frac{17}{4}$

(f) $x + y = 3\sqrt{3}$

$x^2 + xy + y^2 = 21 \Rightarrow (x+y)^2 = 27$
 $+ 6$

45. If $x^2 + y^2 = x^{-2}(777 - y^4)$ and $x + y = x^{-1}(37 - y^2)$, then find the value of $x^2 + y(y - x) = ?$

यदि $x^2 + y^2 = x^{-2}(777 - y^4)$ और $x + y = x^{-1}(37 - y^2)$ तो $x^2 + y(y - x)$ का मान कीजिए?

[A] 29

[B] 21 ✓

[C] 23

[D] 31

$$x^2 + y^2 - xy = \frac{777}{37} = 21$$

$$x^2 + y^2 = \frac{1}{x^2}(777 - y^4)$$

$$x^4 + x^2 y^2 + y^4 = 777$$

$$x^2 + xy + y^2 = 37$$

16. If $x^4 + x^2y^2 + y^4 = 273$ and $x^2 - xy + y^2 = 13$, then the value of xy is?

यदि $x^4 + x^2y^2 + y^4 = 273$ और $x^2 - xy + y^2 = 13$ तो xy का मान है?

[A] 4

[B] 10

[C] 6

[D] 8

$$\frac{21-13}{2}$$

$$x^2 - xy + y^2 = 13 \leftarrow B$$

$$x^2 + xy + y^2 = 21 \leftarrow A$$

47. If $x^4 + x^2y^2 + y^4 = 133$ and $x^2 - xy + y^2 = 7$, then find the value of $(x^2 + y^2)$ is

यदि $x^4 + x^2y^2 + y^4 = 133$ और $x^2 - xy + y^2 = 7$, तो $(x^2 + y^2)$ का मान ज्ञात करें?

[A] 19

[B] 16

[C] 12

[D] 13

$x^2 + xy + y^2 = 19$
 $= 13$

48. If $x^4 + y^4 + x^2y^2 = 117$ & $x^2 + y^2 - xy = 3(4 + \sqrt{3})$ then find $x^2 + y^2$?

यदि $x^4 + y^4 + x^2y^2 = 117$ & $x^2 + y^2 - xy = 3(4 + \sqrt{3})$ है, तो $x^2 + y^2$ ज्ञात कीजिए?

[A] 9

[B] $6\sqrt{3}$

[C] 12 ✓

[D] $13\sqrt{3}$

= 12

$$\begin{array}{r} 39 \\ \cancel{117} \\ \hline 3(4 + \sqrt{3}) \\ \hline 39 \\ \hline 39 \cdot (4 - \sqrt{3}) \\ \hline 12 \end{array}$$

$$x^2 + y^2 + xy = 3(4 - \sqrt{3})$$

49. If $x^2 - xy + y^2 = 17$ and $x^4 + x^2y^2 + y^4 = 425$, then the value of $\frac{x}{y} + \frac{y}{x}$ is

यदि $x^2 - xy + y^2 = 17$ और $x^4 + x^2y^2 + y^4 = 425$ तो $\frac{x}{y} + \frac{y}{x}$ का मान क्या है?

[A] 6.25

[B] 5.25

[C] 6.4

[D] 5.5

$$= \frac{x^2 + y^2}{xy} = \frac{21}{4}$$

50. If $x^2 - xy + y^2 = 7$ and $x^4 + x^2y^2 + y^4 = 105$, then the value of $\frac{x^3}{y^3} + \frac{y^3}{x^3}$ is?

यदि $x^2 - xy + y^2 = 7$ और $x^4 + x^2y^2 + y^4 = 105$ तो $\frac{x^3}{y^3} + \frac{y^3}{x^3}$ का मान क्या है?

[A] $\frac{437}{32}$

[C] $\frac{713}{64}$

[B] $\frac{317}{16}$

[D] $\frac{803}{64}$

$$= k \cdot (k^2 - 3)$$
$$= \frac{11}{4} \times \left(\frac{121}{16} - 3 \right)$$

$$\frac{y}{x} + \frac{x}{y} = \frac{11}{4}$$

51. If $x^4 + x^2y^2 + y^4 = \frac{21}{256}$ and $x^2 + xy + y^2 = \frac{3}{16}$, then $(x + y) = ?$

यदि $x^4 + x^2y^2 + y^4 = \frac{21}{256}$ और $x^2 + xy + y^2 = \frac{3}{16}$ है, तो $(x + y) = ?$

[A] $\frac{1}{16}$

[B] $\frac{5}{8}$

[C] $\frac{3}{8}$

[D] $\frac{1}{4}$

$(x+y)^2 = \frac{1}{16}$

$x^2 + xy + y^2 = \frac{3}{16}$

$x^2 - xy + y^2 = \frac{7}{16}$

$xy = \frac{\frac{3}{16} - \frac{7}{16}}{2} = -\frac{2}{16}$

52. If $x^4 + y^4 + x^2y^2 = 17\frac{1}{16}$ and $x^2 - xy + y^2 = 5\frac{1}{4}$, then one of the values of $(x - y)$ is:

यदि $x^4 + y^4 + x^2y^2 = \frac{273}{16}$ और $x^2 - xy + y^2 = \frac{21}{4}$ तो $(x - y)$ का मान है:

[A] $\frac{5}{2}$
[C] $\frac{5}{4}$

[B] $\frac{3}{4}$
[D] $\frac{3}{2}$

(CGL MAINS 2021)

$$(x - y)^2 = \frac{25}{4}$$

$$x^2 - xy + y^2 = \frac{21}{4}$$

$$x^2 + xy + y^2 = \frac{13}{4}$$

$$xy = \frac{\frac{13}{4} - \frac{21}{4}}{2} = -1$$

54. If $16a^4 + 36a^2b^2 + 81b^4 = 91$ and $4a^2 + 9b^2 - 6ab = 13$, then what is the value of $3ab$?

यदि $16a^4 + 36a^2b^2 + 81b^4 = 91$ और $4a^2 + 9b^2 - 6ab = 13$ है तो $3ab$ का मान क्या है?

[A] -3

[B] 5

[C] $3/2$

[D] $-3/2$

$$-(4a^2 + 9b^2 + 6ab) = 7$$

$$-12ab = 6$$

$$ab = -\frac{1}{2}$$

$$x \rightarrow 2a$$

$$y \rightarrow 3b$$

56. The value of $\frac{(4.6)^4 + (5.4)^4 + (24.84)^2}{(4.6)^2 + (5.4)^2 + 24.84}$ is:

$\frac{(4.6)^4 + (5.4)^4 + (24.84)^2}{(4.6)^2 + (5.4)^2 + 24.84}$ का मान क्या है?

[A] 24.42 $= 4.6^2 + 5.4^2 - 24.84$ **[B] 25.48**

[C] 24.24 $\dots 6+6-4$ **[D] 25.42**

$$\frac{x^4 + y^4 + (xy)^2}{x^2 + y^2 + xy} = x^2 + y^2 - xy$$



53. If $1 + x^2 + x^4 = 585$ and $1 - x + x^2 = 65$, then find the value of $1 + x + x^2$.

यदि $1 + x^2 + x^4 = 585$ और $1 - x + x^2 = 65$ है, तो $1 + x + x^2$ का मान ज्ञात करें।

[A] 8

[B] 7

[C] 6

[D] 9

$$= \frac{585}{65}$$

55. If $x^2 + xy + y^2 = 119$ & $x - \sqrt{xy} + y = 7$, then find xy ?

यदि $x^2 + xy + y^2 = 119$ & $x - \sqrt{xy} + y = 7$ है, तो xy ज्ञात कीजिये?

[A] 25

[B] 5

[C] 50

[D] 36

$$\begin{array}{r} x - \sqrt{xy} + y = 7 \\ - \\ x + \sqrt{xy} + y = 17 \\ \hline \end{array}$$

$$\sqrt{xy} = 5$$

$$xy = 25$$

$$x^4 + x^2 y^2 + y^4 = (x^2 + xy + y^2) \cdot (x^2 - xy + y^2)$$

$$\begin{aligned} x &\rightarrow x \\ y &\rightarrow 1 \end{aligned}$$

$$x^4 + x^2 + 1 = (x^2 + x + 1) \cdot (x^2 - x + 1)$$

$$\begin{aligned} x &\rightarrow \sqrt{x} \\ y &\rightarrow \sqrt{y} \end{aligned}$$

$$x^2 + xy + y^2 = (x + \sqrt{xy} + y) \cdot (x - \sqrt{xy} + y)$$



57. Using algebraic identities, simplify the following expression. $\frac{(x^4 + x^2 + 1)}{(x^2 + x + 1)}$

बीजगणितीय सर्वसमिका (algebraic identities) का उपयोग करते हुए, निम्न व्यंजक को सरल करें। $\frac{(x^4 + x^2 + 1)}{(x^2 + x + 1)}$

[A] $(x^2 - 2x + 1)$

[B] $(x^2 + x + 1)$

[C] $(x^2 + 2x + 1)$

[D] $(x^2 - x + 1)$



58. If $9x^4 + 20x^2y^2 + 16y^4 = 91$ and $3x^2 + 2xy + 4y^2 = 13$, then find the value of $\frac{x}{4y} + \frac{y}{3x} = ?$

यदि $9x^4 + 20x^2y^2 + 16y^4 = 91$ और, $3x^2 + 2xy + 4y^2 = 13$ तो $\frac{x}{4y} + \frac{y}{3x}$ का मान ज्ञात करें?

[A] $\frac{7}{12}$
[C] $\frac{2}{3}$

[B] 0.625
[D] $\frac{5}{9}$

$$= \frac{3x^2 + 4y^2}{12xy} = \frac{10}{18}$$

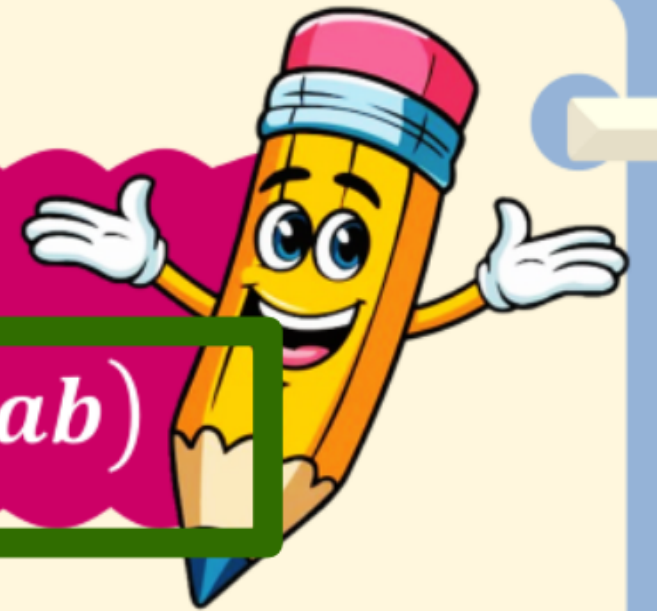
$$3x^2 + 2xy + 4y^2 = 13$$

$$3x^2 - 2xy + 4y^2 = 7$$

$$4xy = 6$$

$$a^4 + b^4 = (a^2 + b^2)^2 - (\sqrt{2}ab)^2$$

$$a^4 + b^4 = (a^2 + b^2 + \sqrt{2}ab)(a^2 + b^2 - \sqrt{2}ab)$$



$$a^4 + b^4 = (a^2 + b^2)^2 - 2a^2b^2$$

याद करो

59. If $a^2 + b^2 = x$, $ab = y$, $\frac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2} = ?$

यदि $a^2 + b^2 = x$, $ab = y$, तो $\frac{a^4 + b^4}{a^2 - ab\sqrt{2} + b^2} = ?$

[A] $x + 2y$

[C] $y + \sqrt{2}x$

[B] $x + \sqrt{2}y$

[D] $2x + y$

$(a^2 + b^2) + \sqrt{2}ab$
 $= x + \sqrt{2}y$

60. If $a^4 + b^4 = 63$ & $a^2 + b^2 + ab\sqrt{2} = 9$, then find ab ?

यदि $a^4 + b^4 = 63$ & $a^2 + b^2 + ab\sqrt{2} = 9$ है, तो ab ज्ञात कीजिये?

[A] 1

[B] $1/\sqrt{2}$

[C] 2

[D] 0

$$\cancel{ab\sqrt{2}} = \cancel{2}$$

$$ab = \frac{1}{\sqrt{2}}$$

61. If the semi-perimeter and area of a rectangular whose length and breadth are 'x' and 'y' is 12 cm and 28 cm², respectively, then find the value of $x^4 + x^2y^2 + y^4$.

यदि उस आयताकार मैदान की अर्ध-परिधि और क्षेत्रफल क्रमशः 12 cm और 28 cm² है, जिसकी लंबाई 'x' और चौड़ाई 'y' हो, तो $x^4 + x^2y^2 + y^4$ का मान ज्ञात कीजिए।

SSC Selection Post (Phase-XII)

[A] 6609

[B] 6906

[C] 6960

[D] 6690

$$(x^2 + y^2 + xy) \cdot (x^2 + y^2 - xy) = 116 \times 60$$



$$\begin{aligned} (x+y) &= 12 \\ xy &= 28 \end{aligned}$$

$$x^2 + y^2 = 12^2 - 2 \times 28 = 88$$



62. If $x^4 + x^3 + x^2 + x + 1 = 0$, then $x^{2015} + x^{3015} + 5 = ?$

यदि $x^4 + x^3 + x^2 + x + 1 = 0$ है, तो $x^{2015} + x^{3015} + 5 = ?$

[A] 6

[B] 7 $1+1+5$

[C] 8

[D] -9

$$\begin{aligned} x^4 + x^3 + x^2 + x + 1 &= 0 \\ x(x^4 + x^3 + x^2 + x + 1) &= 0 \cdot x \\ x^5 + x^4 + x^3 + x^2 + x &= 0 \end{aligned}$$

$$x^5 - 1 = 0$$

$$x^5 = 1$$

63. If $a^3 + 4a^2 + 16a = 1$, then what is the value of $a^3 + \left(\frac{4}{a}\right)$?

यदि $a^3 + 4a^2 + 16a = 1$ है, तो $a^3 + \left(\frac{4}{a}\right)$ का मान क्या है?

[A] 63

[B] 65

[C] 67

[D] 68

= 65

$$\begin{array}{r} a^3 + 4a^2 + 16a = 1 \\ \times \frac{4}{a} \\ \hline 4a^2 + 16a + 64 = \frac{4}{a} \\ \hline a^3 - 64 = 1 - \frac{4}{a} \end{array}$$

64. If $a^3 + 5a^2 + 25a + 2 = 0$, then find the value of $a^3 - \frac{10}{a}$?

यदि $a^3 + 5a^2 + 25a + 2 = 0$ है, तो $a^3 - \frac{10}{a}$ का मान ज्ञात कीजिये?

[A] 127

[B] 125

[C] 25

[D] 123

H.W